CCSU Integration Bee, March 31, 2023

1. Only a writing implement will be allowed. No calculators, formula sheets, or extra paper are permitted. Scrap paper will be provided.

2. There will be three rounds.

3. All integrals will be indefinite.

4. Students do not have to write the arbitrary constant $(+C)$ to get a correct solution. They must, however, undo all substitutions and thus present a solution in terms of the original variable.

5. The integrals will tend to increase in difficulty from round to round, and within each round as the round progresses.

6. An answer is acceptable as long as it is a solution for the original problem on an open interval (any open interval). So for example, $\ln(x)$ is an acceptable solution to $\int \frac{1}{x}\,dx$ because it is a solution on $(0, \infty)$.

7. A 10-second leeway will be given on time.

8. Check Rule: Students during the competition can ask the judge for a "check" at any time ONLY ONCE (relinquishing the ticket provided when you do so). The judge will offer one of the following non-verbal responses:

   (a) If the answer is correct: a check mark ✓. Congratulations!

   (b) If there is a mistake that changes the nature of the integral or the technique is entirely off and requires replacing significant portions of your work: an ×. This means you have major errors. Try to fix them!

   (c) If there is a mistake that does not change the nature of the integral and could be fixed by changing your work in one or just a few spots: a circle mark ∘. This means you have minor error(s). Try to fix them!

   (d) If the judge is unable to ascertain (for whatever reason) if the error is major or minor: a ?. This response is unlikely. Try to assess your own work and keep going!

   YOU MUST HAVE A CIRCLED ANSWER FOR THE JUDGE TO EXAMINE. Since time is of the essence you should ask for your check by raising your hand well before time runs out. Once you raise your hand you cannot change your mind. You can do this only once.

   Here are some examples illustrating the sample responses:

$$\int \frac{1}{\sqrt{x^2 - 1}}\,dx = \ln\left(\sqrt{x^2 - 1} + x\right), \quad ✓$$

$$\int \frac{1}{\sqrt{x^2 - 1}}\,dx = \int \frac{1}{x - 1}\,dx = \ln|x - 1|, \quad ×$$

$$\int \frac{1}{\sqrt{x^2 - 1}}\,dx = \ln|\sec(t) + \tan(t)|, \quad ∘$$

9. Round 1: Three integrals will be written on the board. All participants will, at their seats, have 5:00 minutes to compute the integrals. Each student must get at least two of the three integrals correct to advance to Round 2.

10. Round 1a: If there are more than 16 participants who have passed through Step 1, one Round 2 integral will be written on the board and the survivors of the first cut will have 4:00 minutes to compute it correctly at their seats. A participant will be eliminated if his/her solution is incorrect. This will repeat until there are no more than 16 participants left. Every student who advances to Round 2 will receive a $5 Starbucks voucher!

11. Round 2: The students, in a randomly-chosen order, will be asked in sets of four to do the same integral at the board (they will not be able to see each other's work), and will have 4:00 minutes to compute this integral. This is a two-strike round; i.e. a player will be eliminated when they have gotten two integrals incorrect. We will cycle (in order) through the participants until the number of remaining participants has been reduced to at most four, making sure that the last cycle of students has been completed.

12. Round 3: The four (or fewer) participants in this round will all receive a major prize. The students, in randomly-chosen order, will compute an integral at the board with a time limit of 5:00 minutes. Each student will be eliminated after two incorrect answers (two strikes), and the round will continue until a winner is crowned.

Sample Round Questions

Round 1: $\int 2xe^x \, dx$

Round 2: $\int \dfrac{e^x}{\sqrt{1-e^x}} \, dx$

Round 3: $\int e^x \cos^2(e^x) \, dx$

These represent the average level of difficulty of the questions for each round. All standard techniques from a Calculus II curriculum (u-substitution, integration by parts, trigonometric substitutions, partial fractions decomposition) are fair game.